Evaluating Derivatives

At a point,

Finding the slope of the curve
Finding the slope of the tangent line
Finding the instantaneous rate of change

Find the slope of the tangent line to

$y = e^{\cos x}$ at $x = \pi$

$y' = e^{\cos x}(-\sin x)$

$f'(\pi) = e^{\cos \pi}(-\sin \pi)$

$= e^{-1}(0)$

$m = 0$

Find the instantaneous rate of change of

$f(t) = e^{-t}\,t^2$ at $t = 2$

$$f'(x) = -e^{-t}\cdot t^2 + e^{-t}\cdot 2t$$

$$= -t^2 e^{-t} + 2te^{-t}$$

$$f'(2) = \frac{-4}{e^2} + \frac{4}{e^2} = 0$$

$$\frac{d}{d\boxed{x}}(f(x)\Big|_{x=\boxed{2}}$$

Write the equation of the tangent line and normal line to $y = 2x^2 - 3x$ at $x = 4$

$m = 13$

Tangent.

$(4, 20)$

Find the linear approximation of $y = 2x^2 - 3x$ at $x = 4$.

Use it to estimate $f'(4.1)$

An orange farmer currently has 200 trees yeilding an average of 15 bushels of oranges per tree. She is expanding her farm at a rate of 15 trees per year, while improved husbandry is improving her average annual yield by 1.2 bushels per tree. What is the current (instantaneous) rate of increase of her total annual production of oranges?

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Numbers 17,18,31,32

pages 124 - 126

Numbers 37-42,47,~~51~~,~~52~~

Instantaneous Velocity

The instantaneous velocity is the derivative of the position function $s = f(t)$ with respect to time. At time t, the velocity is

$$v(t) = \frac{ds}{dt}$$

a.) Find the rate of change of the area A of a circle with respect to its radius r.

b.) Evaulate the rate of change of A at r = 5 and at r = 10.

c.) If r is measured in inches and A is measured in square inches, what units would be appropriate for dA/dr?

Suppose that an object is moving along a coordinate line so that we know its position s on the line as a function of time t:

$$s = f(t)$$

The displacement of the object over the time interval from t to t + Δ t is (final pos - initial pos)

$$\Delta s = f(t + \Delta t) - f(t)$$

The average velocity of the object over that time interval is

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

$$= \frac{\Delta \text{position}}{\Delta \text{time}}$$

Velocity = 0 means particle is at rest

If velocity = 0,
acceleration is not necessarily = 0

If velocity and acceleration have the same sign, then particle is speeding up

If velocity and acceleration have opposite signs, then the particle is slowing down

$x(t) = -t^3 + 7t^2 - 14t + 8$ meters

1.) Find displacement over the first 3 seconds.

2.) Find the average velocity over time 0 to 3 sec.

3.) Find velocity at $t = 2$ seconds.

Speed

Speed is the absolute value of velocity

$$\text{Speed} = |v(t)| = \left|\frac{ds}{dt}\right|$$

Acceleration

Acceleration is the derivative of velocity with respect to time. If a body's velocity at time is $v(t) = ds/dt$, then the body's acceleration at time t is

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Free Fall Constants

English units : $g = 32 \text{ ft/sec}^2$, $s = 1/2(32)t^2 = 16t^2$

s in feet from $1/2\ gt^2$

Metric units : $g = 9.8 \text{ m/sec}^2$, $s = 1/2(9.8)t^2 = 4.9t^2$

s in meters

A dynamite blast propels a heavy rock straight up with a launch velocity of 160 ft/sec (about 109 mph). It reaches a height of $s = 160t - 16t^2$ after t seconds.

a.) How high does the rock go?

b.) What is the velocity and speed of the rock when it is 256 feet above the ground on the way up? on the way down?

c.) What is the acceleration of the rock at any time t during its flight (after the blast)?

d.) When does the rock hit the ground?

A particle moves along a line so that its position at any time $t \geqslant 0$ is given by the function

$s(t) = t^2 - 4t + 3$, where s is measured in meters and t is measured in seconds.

a.) Find the displacement of the particle during the first two seconds.

b.) Find the average velocity of the particle during the first 4 seconds

c.) Find the instantaneous velocity of the particle when $t = 4$.

d.) Find the acceleration of the particle when $t = 4$.

e.) Describe the motion of the particel. At what values of t does the particle change direction?

Find acceleration of the particle at times when it is at rest. **(At rest means velocity is zero)**

$$x(t) = -t^3 + 7t^2 - 14t + 8 \text{ meters}$$

$v(t) =$

Given $x(t) = 2t^3 - 13t^2 + 22t - 5$ m/sec

1.) Find the initial position

2.) Find the displacement after 3 seconds

3.) Find the acceleartion at times when the particle is at rest.

4.) Find the average velocity over the first 3 seconds.

5.) At $t = 5$, is the particle moving toward or away from the origin?

page 137, numbers 19, 21, 23, 40 - 45

Derivatives in Economics

Cost Production = $c(x)$ → Number of units produced

Marginal Cost of Production = rate of change of cost with respect to the level of production a.k.a. dc/dx

Suppose it costs

$$c(x) = x^3 - 6x^2 + 15x$$

dollars to produce x radiators when 8 to 10 radiators are produced, and that

$$r(x) = x^3 - 3x^2 + 12x$$

gives the dollar revenue from selling x radiators. Your shop currently produces 10 radiators a day. Find the marginal cost and marginal revenue.